Bayesian Inference for Sample Surveys

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Distinctive features of survey inference

1. Primary focus on descriptive finite population quantities, like overall or subgroup means or totals
   - Bayes – which naturally concerns predictive distributions -- is particularly suited to inference about such quantities, since they require predicting the values of variables for non-sampled items
   - This finite population perspective is useful even for analytical quantities:

\[
\theta = \text{model parameter (meaningful only in context of the model)}
\]

\[
\tilde{\theta}(Y) = "\text{estimate}" \text{ of } \theta \text{ from fitting model to whole population } Y
\]

(a finite population quantity, exists regardless of validity of model)

A good estimate of \( \theta \) should be a good estimate of \( \tilde{\theta} \)

(if not, then what's being estimated?)
Distinctive features of survey inference

2. Analysis needs to account for "complex" sampling design features such as stratification, differential probabilities of selection, multistage sampling.

- Samplers reject theoretical arguments suggesting such design features can be ignored if the model is correctly specified.
- Models are always misspecified, and model answers are suspect even when model misspecification is not easily detected by model checks (Kish & Frankel 1974, Holt, Smith & Winter 1980, Hansen, Madow & Tepping 1983, Pfeffermann & Holmes (1985).
- Design features like clustering and stratification can and should be explicitly incorporated in the model to avoid sensitivity of inference to model misspecification.
Distinctive features of survey inference

3. A production environment that precludes detailed modeling.

- Careful modeling is often perceived as "too much work" in a production environment (e.g. Efron 1986).
- Some attention to model fit is needed to do any good statistics.
- “Off-the-shelf" Bayesian models can be developed that incorporate survey sample design features, and for a given problem the computation of the posterior distribution is prescriptive, via Bayes Theorem.
- This aspect would be aided by a Bayesian software package focused on survey applications.
Distinctive features of survey inference

4. Antipathy towards methods/models that involve strong subjective elements or assumptions.
   - Government agencies need to be viewed as objective and shielded from policy biases.
   - Addressed by using models that make relatively weak assumptions, and noninformative priors that are dominated by the likelihood.
   - The latter yields Bayesian inferences that are often similar to superpopulation modeling, with the usual differences of interpretation of probability statements.
   - Bayes provides superior inference in small samples (e.g. small area estimation)
Distinctive features of survey inference

5. Concern about repeated sampling (frequentist) properties of the inference.

• Calibrated Bayes: models should be chosen to have good frequentist properties
• This requires incorporating design features in the model (Little 2004, 2006).
Survey Inference Setup

\( Z = (Z_1,\ldots,Z_N) \) = design variables, known for population

\( Y = (Y_1,\ldots,Y_N) \) = population values, recorded only for sample

\( Q = Q(Y,Z) \) = target finite population quantity

\( I = (I_1,\ldots,I_N) \) = Sample Inclusion Indicators

\( I_i = \begin{cases} 
1, & \text{unit included in sample} \\
0, & \text{otherwise} 
\end{cases} \)

\( Y_{\text{inc}} = Y_{\text{inc}}(I) \) = part of \( Y \) included in the survey

\( Y = (Y_{\text{inc}},Y_{\text{exc}}) \)
Models

• Joint distribution of \((Y, I)\) conditional on \(Z\)

• Two approaches

\[
\begin{align*}
\Pr(Y, I \mid Z) & = \Pr(Y \mid Z) \Pr(I \mid Y, Z) \\
\Pr(Y, I \mid Z) & = \Pr(Y \mid I, Z) \Pr(I \mid Z)
\end{align*}
\]

• Typically

(Sampling mechanism does not depend on the survey outcomes)

\[
\Pr(I \mid Y, Z) = \Pr(I \mid Z)
\]

(Same substantive model applies to both sampled and nonsampled Subjects)

\[
\Pr(Y \mid I, Z) = \Pr(Y \mid Z)
\]
Model Specification

• Indices used to identify subjects in the population (conditional on \( Z \)) is assumed to be arbitrary.

• Exchangeable joint distribution

\[
\Pr(Y_1, Y_2, \cdots, Y_N \mid Z) = \Pr(Y_{i_1}, Y_{i_2}, \cdots, Y_{i_N} \mid Z)
\]

\((i_1, i_2, \cdots, i_N)\) is a permutation of \((1, 2, \cdots, N)\).

• Exchangeable distribution are of the form

\[
Y_i \mid Z, \theta \sim \text{independent}
\]

\[
\pi(\theta) = \text{prior}
\]
Examples

• Assume SRS and no $Z$, binary $Y$
  \[ Y_i \mid \theta \sim \text{iid Bern}(1, \theta), i = 1, 2, \ldots, N \]
  \[ \theta \sim \text{Beta}(a, b); a, b \text{ known} \]

• $Z$: $H$ Strata, SRS within stratum, Continuous $Y$
  \[ Y_{ih} \mid Z = h \sim \text{iid } N(\mu_h, \sigma^2_h) \]
  \[ \pi(\mu_h, \log \sigma_h) \sim \text{BVN} \]

• Cluster sampling, Count $Y$
  \[ Y_{ic} \sim \text{iid Poisson}(\lambda_c), i = 1, 2, \ldots, N_c \]
  \[ \log \lambda_c \sim \text{iid } N(\mu, \sigma^2), c = 1, 2, \ldots, C \]
  \[ \pi(\mu, \log \sigma) \sim \text{BVN} \]
Inference

- Observed data: \( \{Y_{inc}, Z, I\} \)
- Unobserved or missing data: \( Y_{exc} \)
- Model: \( \Pr(Y \mid Z) \)
- Inference: \( \Pr(Y_{exc} \mid Z, I, Y_{inc}) \)
- Goal: Simulate copies of \( Y_{exc} \) by drawing from the above predictive distribution and compute the estimand of interest \( Q(Y,Z) \)
- Multiple Imputation of Missing Values or create synthetic populations
Example

• Housing and Children Study to evaluate the effect of providing housing voucher on child development
• Population: All applicants for voucher
• Treatment: Random Selection
• Control: Rest of the population
• Survey: Samples of Treatment and Control subjects
• Two waves, Dried Blood spots, Child development measures, adult primary care giver
Data Setup

• Z: Data from sampling frame (from voucher application)
• T for Treatment and C for Control
• Y(T): Measures for Treatment subjects
• Y(C): Measures for Control Subjects
Fill-in Synthetic potential populations
Inference

• Create several potential synthetic populations under treatment and control conditions
• Compute summary measures (such as mean, median etc.)
• Compare the distribution of summary measures under treatment and control conditions
  – Numerical summaries
  – Graphical summaries like histogram or kernel densities
• Analyze the two sets of populations to discern treatment effects, heterogeneity of treatment effects etc.
Summary

• Bayes inference for surveys must incorporate design features such as stratification, weighting and clustering appropriately.
• Bayes inference is not asymptotic, and delivers good frequentist properties in small samples.
• Software like BUGS (PROC MCMC in SAS) can be used to implement fully model based framework.
• Recasting the Bayesian inference problem as missing data problem allows the use of multiple imputation software.
• Nonparametric Bayes allows incorporation of complex design features without making strong model assumptions.
• Pseudo or synthetic population framework makes the inference problem easy (just compute any estimand of interest).
• Give it a try!! (you will love it 😊)