Using Instrumental Variables Analysis to Investigate Mediation Processes in Multisite Randomized Trials

Sean F. Reardon
Stanford University

based on collaborative work with

Stephen W. Raudenbush
Howard Bloom

Outline

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4. Formal Assumptions of the MSMM-IV design
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<u>A Motivating Example:</u>

Suppose we offer families a voucher to pay for up to $5,000 for pre-school. We want to know if this improves children's school readiness.

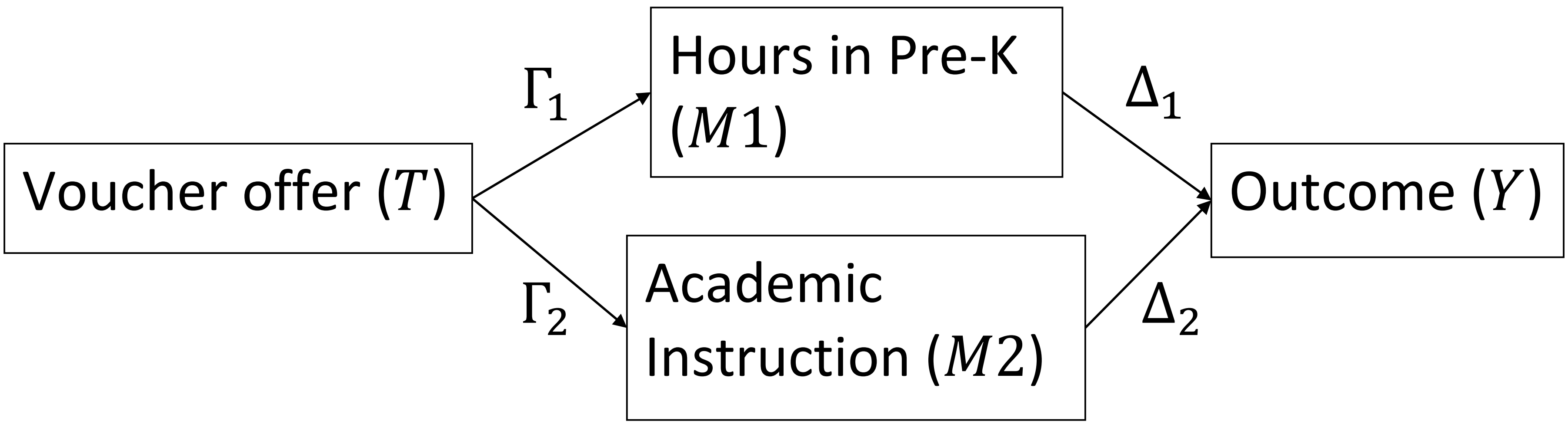
Under random assignment, it is straightforward to estimate the average effect of the voucher offer on school readiness.

In a multi-site trial, it may also be possible to estimate the variance in the effect of the voucher across sites.

But if we wish to know <u>how</u> the voucher affects readiness, **we need a theory, and a design that allows us to test that theory.**

A simple (simplistic) theory:

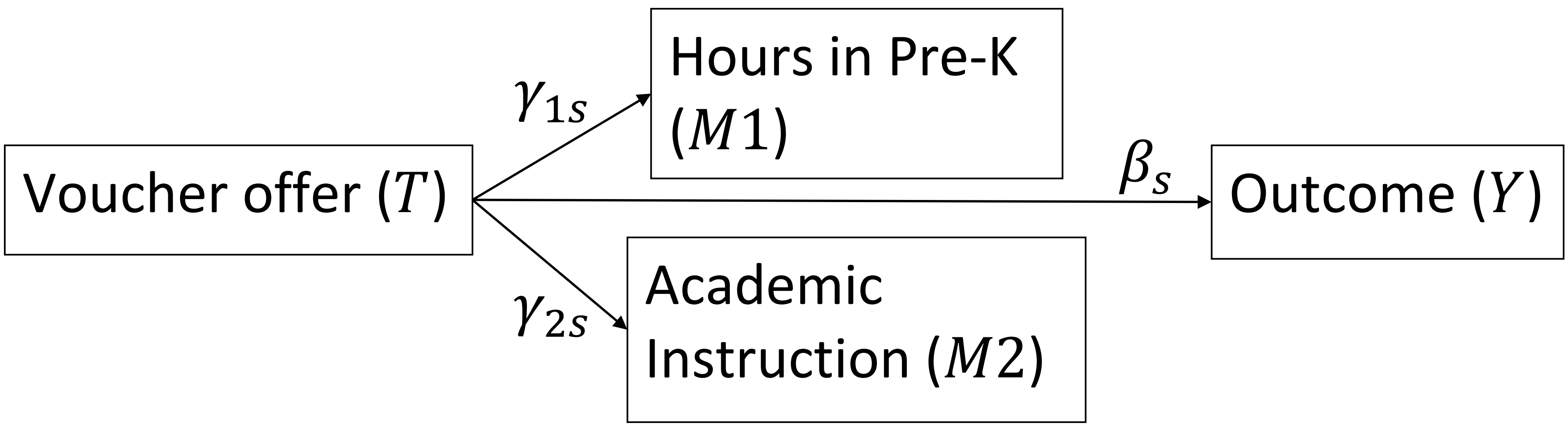
The voucher offer may induce a) more time in pre-school and/or b) the use of pre-K programs that focus on teaching academic skills.



How do we test this theory, and estimate the effects of pre-school time and type?

Suppose we conduct this experiment in a number of sites. In each site s we can estimate both

a) The average effect of the voucher offer on school readiness (call it β_s).
b) The average effects of the voucher offer on both time and type of pre-school (call these γ_{1s} and γ_{2s}).



Next we fit the regression model associating the effects of the offer on the mediators to its effect on school readiness:

$$\beta_s = \delta_1(\gamma_{1s}) + \delta_2(\gamma_{2s}) + \omega_s$$

The logic here is that β_s will be bigger in sites where the voucher offer had bigger effects on the mediators, all else being equal.

Here δ_1 is the association between the effect of the voucher offer on readiness (β_s) and its effect on children's time in pre-K (γ_{1s}), holding constant its effect on pre-K type (γ_2).

Under some assumptions (described below), δ_1 is the average effect of additional time in pre-K on school readiness.

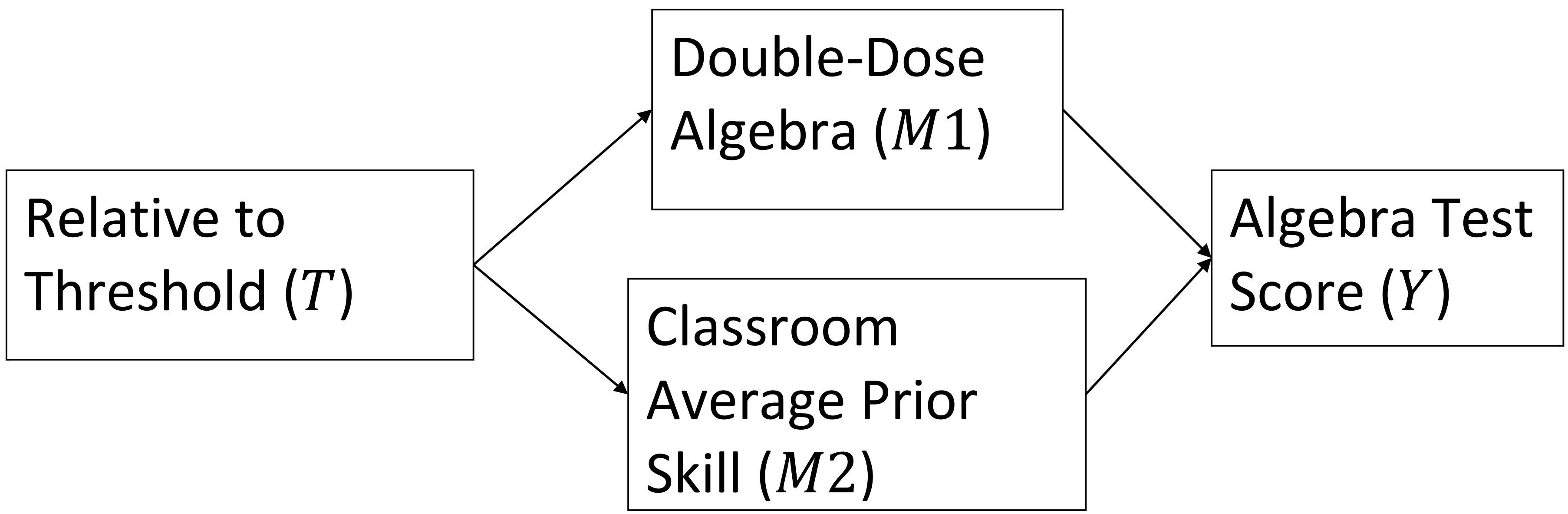
Example 2:

Nomi & Raudenbush (2013) estimate the effect of Chicago's "double-dose" algebra program, in which students scoring below a threshold were assigned to two periods of algebra rather than one.

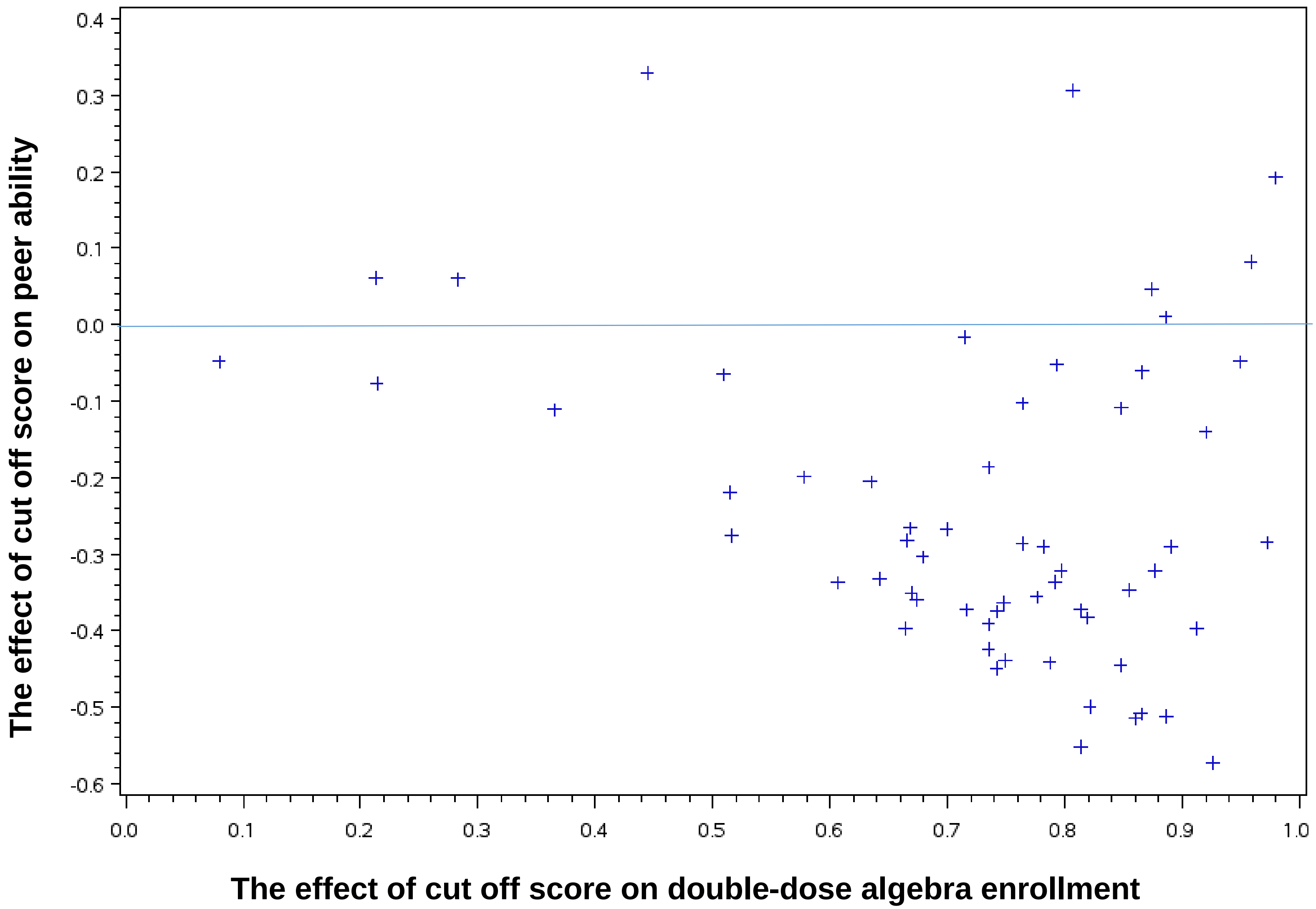
The RD design was implemented in 60 schools (a multi-site RD design).

Nomi & Raudenbush argue that the assignment to double dose affected both a) the amount of algebra instruction students received and b) the peer composition of their algebra classes. The multi-site nature of the design allows them to investigate the effects of both peer composition and algebra "dosage" on student learning.

Double-Dose Algebra Effects (Nomi & Raudenbush)



Joint distribution of the compliance effects on double-dose algebra enrollment and peer ability



Estimated Effects of Double-Dose Algebra and Peer Ability

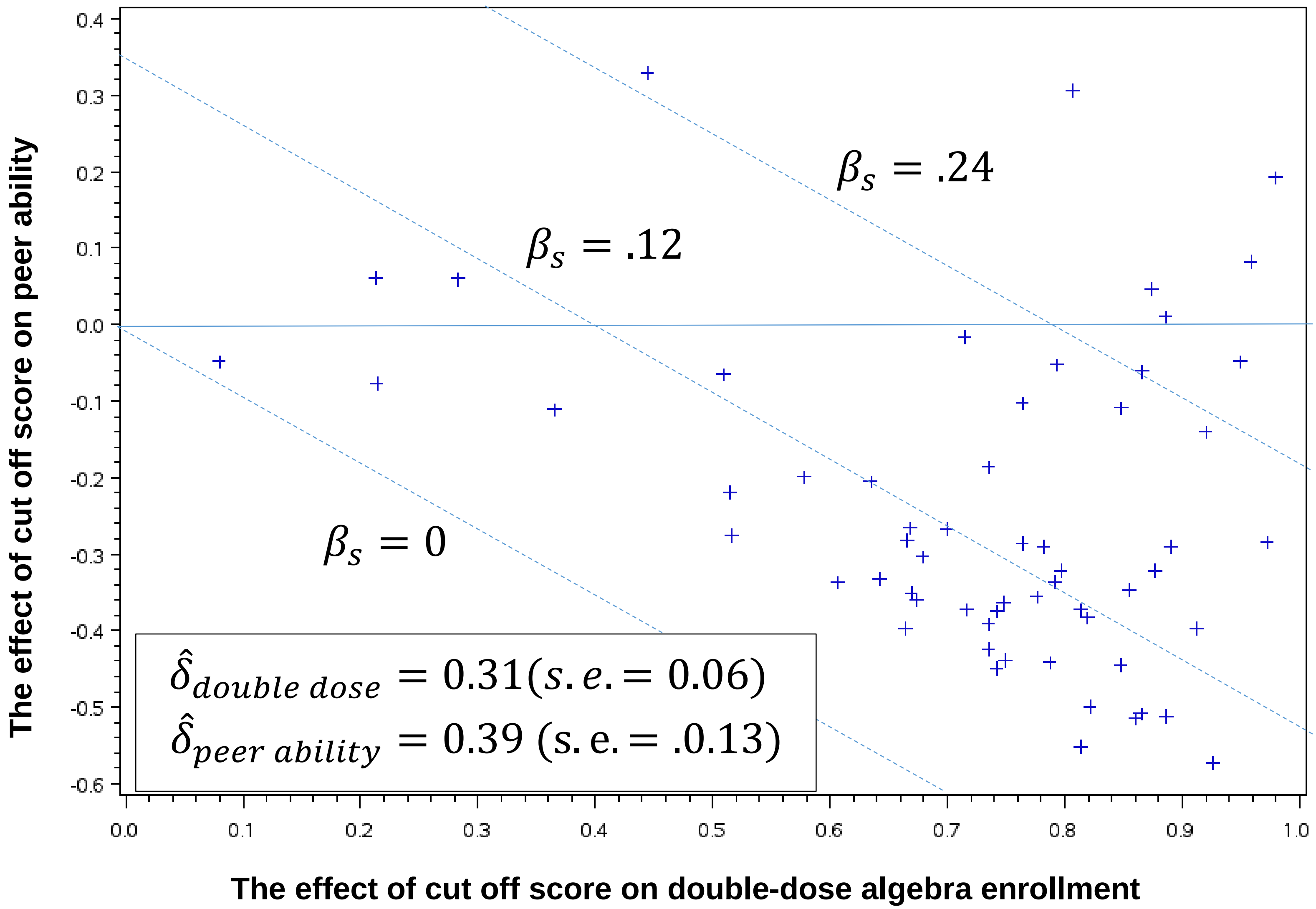
The MSMM-IV model is equivalent to fitting:

$$\hat{\beta}_s^{RD} = \delta_{dd}\left(\hat{\gamma}_{(double\ dose)s}^{RD}\right) + \delta_{pa}\left(\hat{\gamma}_{(peer\ ability)s}^{RD}\right) + e_s$$

via WLS (where the weights are inversely proportional to the square of $se(\hat{\beta}_s^{RD})$. We get:

$$\hat{\delta}_{dd} = 0.31 \ \ (se = 0.06)$$
$$\hat{\delta}_{pa} = 0.39 \ \ (se = 0.13)$$

Estimated effects of double-dose algebra enrollment and peer ability on algebra scores



Example 3:

Duncan, Morris, Rodrigues (2011) use sixteen implementations of random-assignment welfare-to-work experiments (each with slightly different program designs) to estimate the impact of three hypothesized mediators of the programs: income, hours worked, and welfare receipt.

Within any site, it is impossible to use IV to estimate the effects of 3 mediators based on a single instrument (random assignment to program). But the replication of the study across 16 sites enables them to generate 16 instruments (site-by-assignment interactions) to estimate the effect of 3 hypothesized mediators.

Example 4:

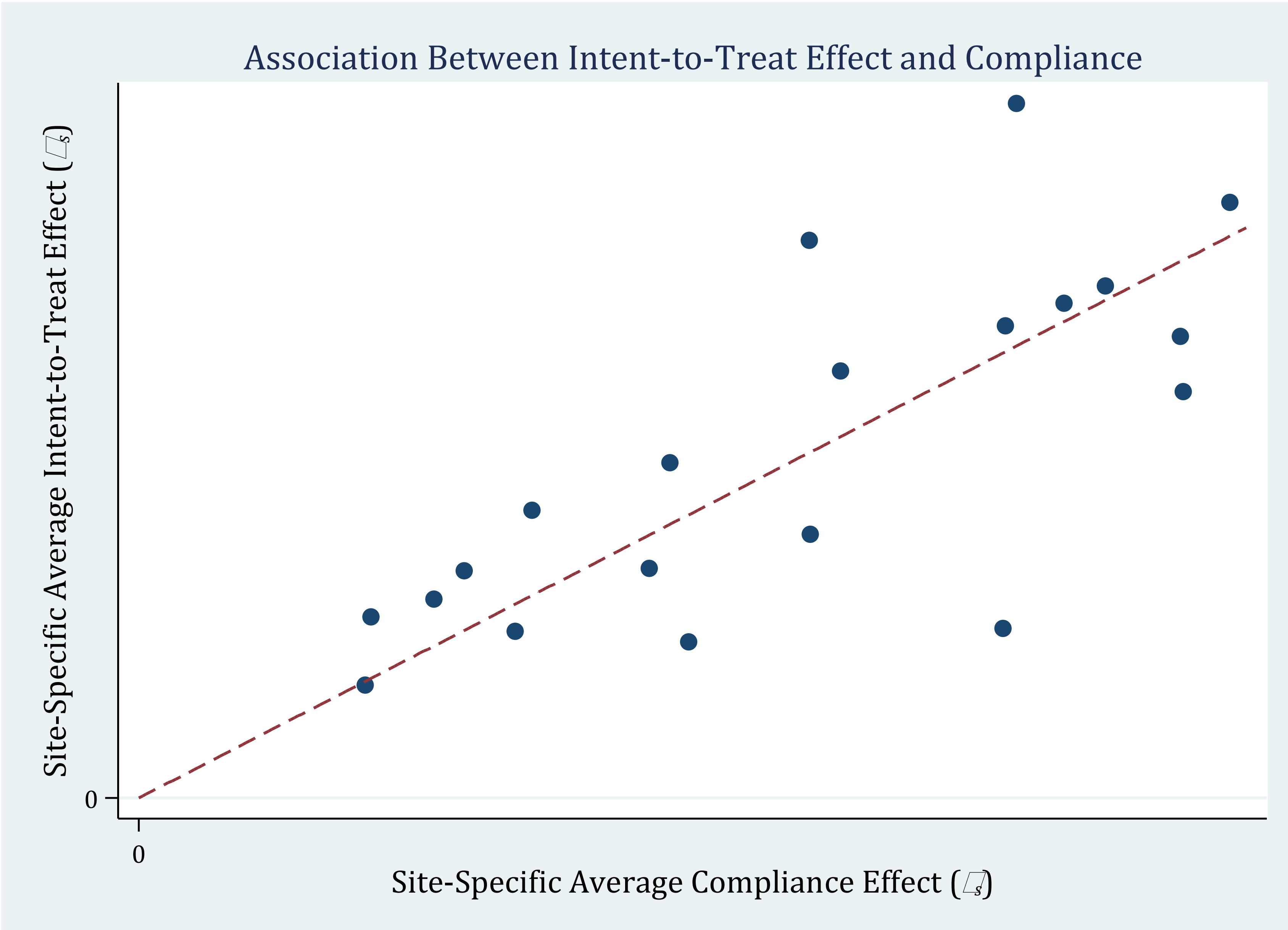
In 5 cities, the MTO experiment randomly assigned low-income families to receive 1) a housing voucher to pay for housing in a low-poverty neighborhood; 2) a regular section 8 housing voucher; or 3) no voucher.

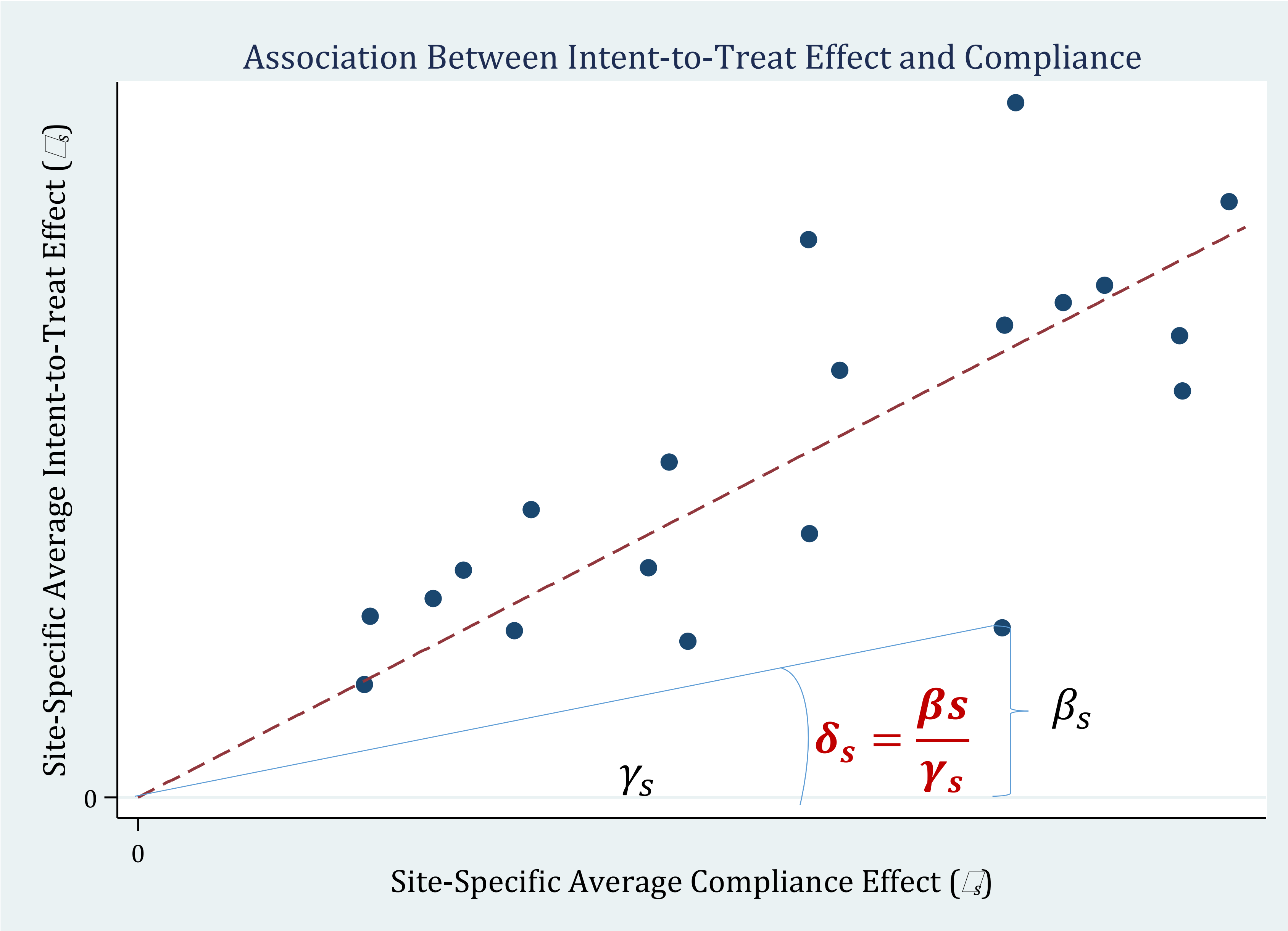
Kling, Liebman, and Katz (2007) use the study to estimate the effects of neighborhood poverty on child/family outcomes, controlling for the effect of moving (using the voucher). They rely on 10 instruments (5 sites interacted with 2 treatment variables) to identify the effect of the two mediators (neighborhood poverty and mobility).

The MSMM-IV Assumptions

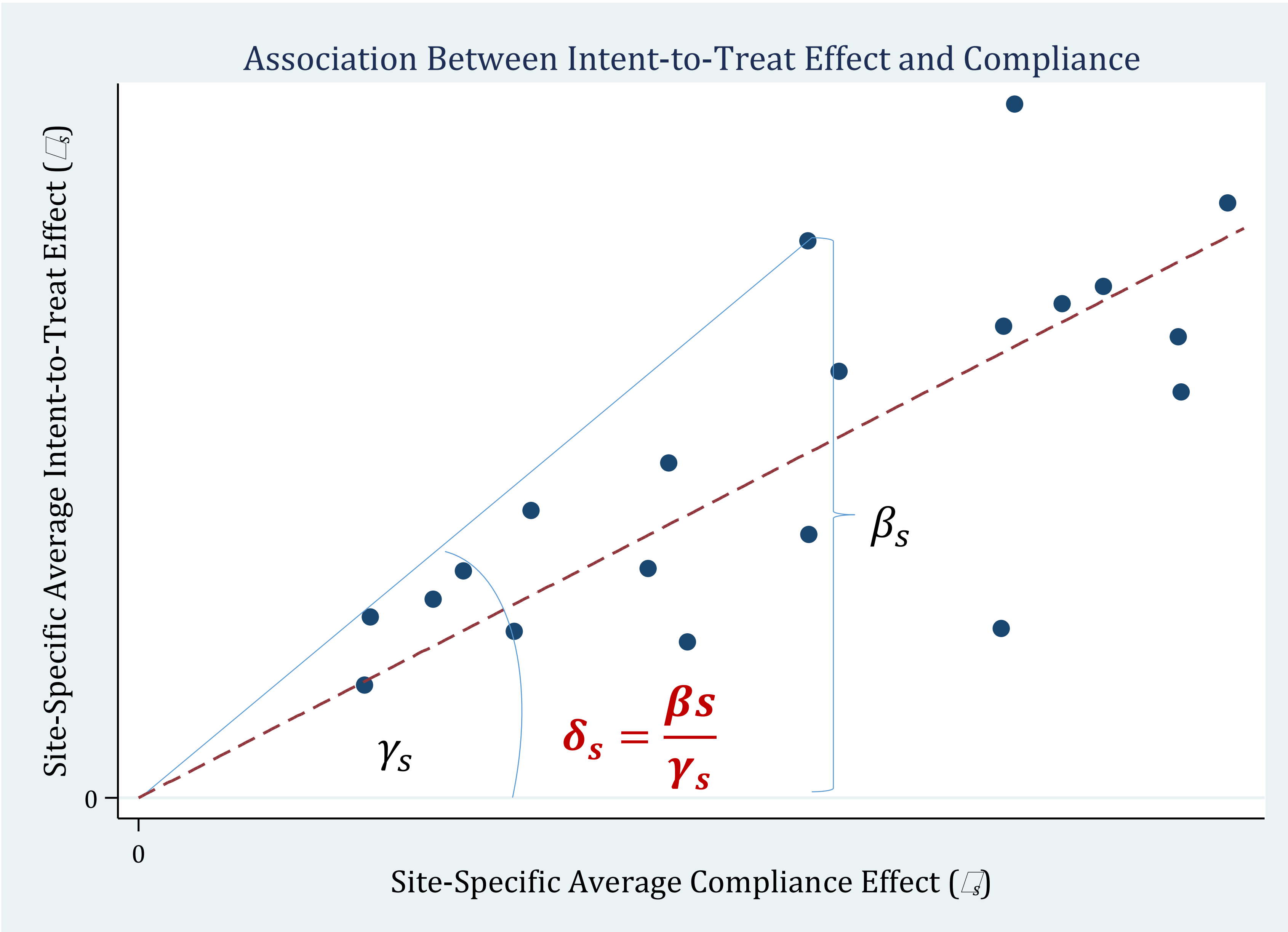
1) Stable unit treatment value assumptions
2) Person-specific linearity of the mediators with respect to the treatment
3) Person-specific linearity of the outcome with respect to the mediators
4) **Parallel mediators**
5) **Exclusion restriction**
6) Zero within-site compliance-effect covariance for each mediator
7) Within-site ignorable treatment assignment
8) Compliance matrix has sufficient rank
9) **Between-site cross-mediator compliance-effect independence**

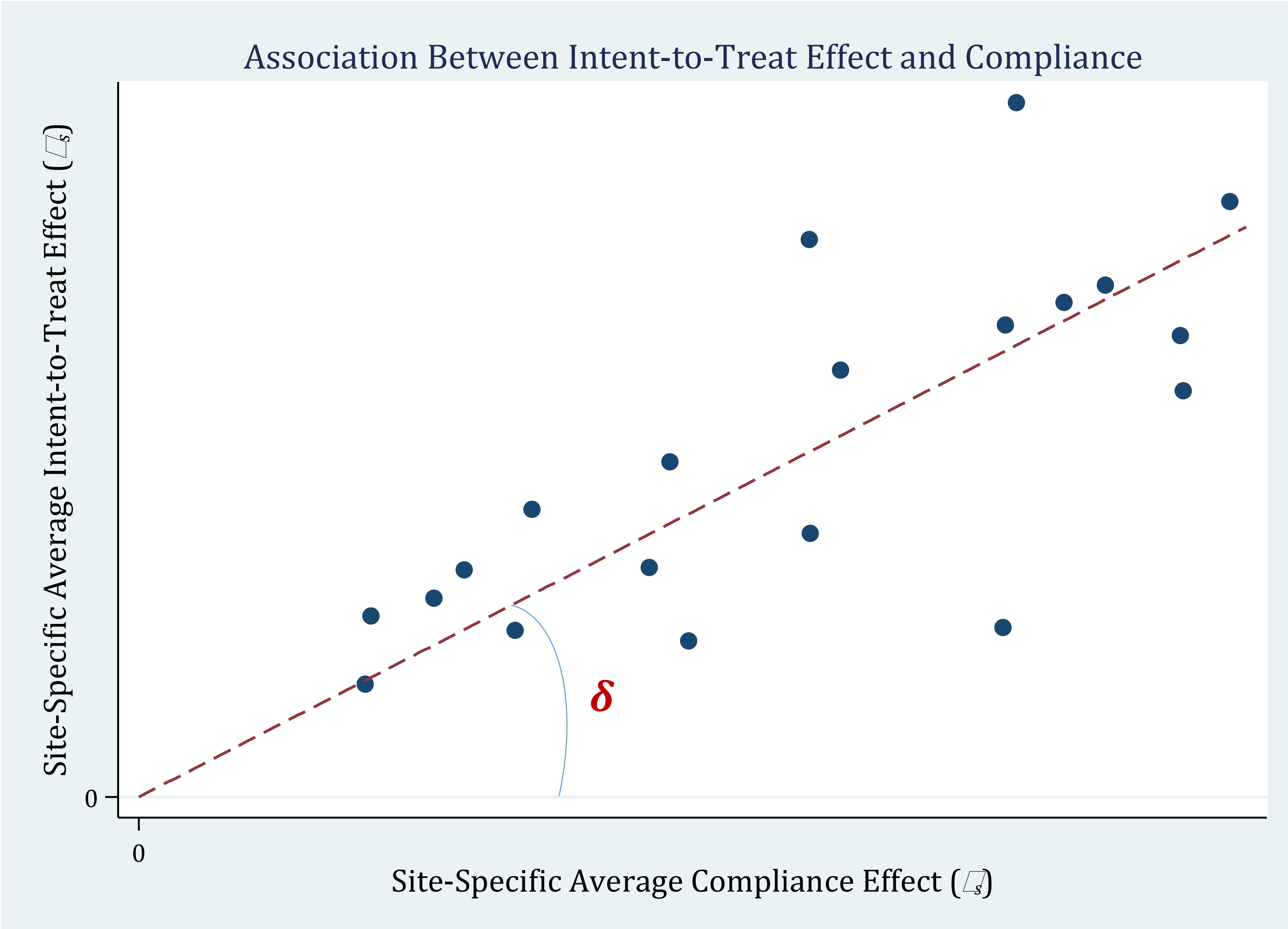
Visualizing the MSMM-IV model with a single mediator:

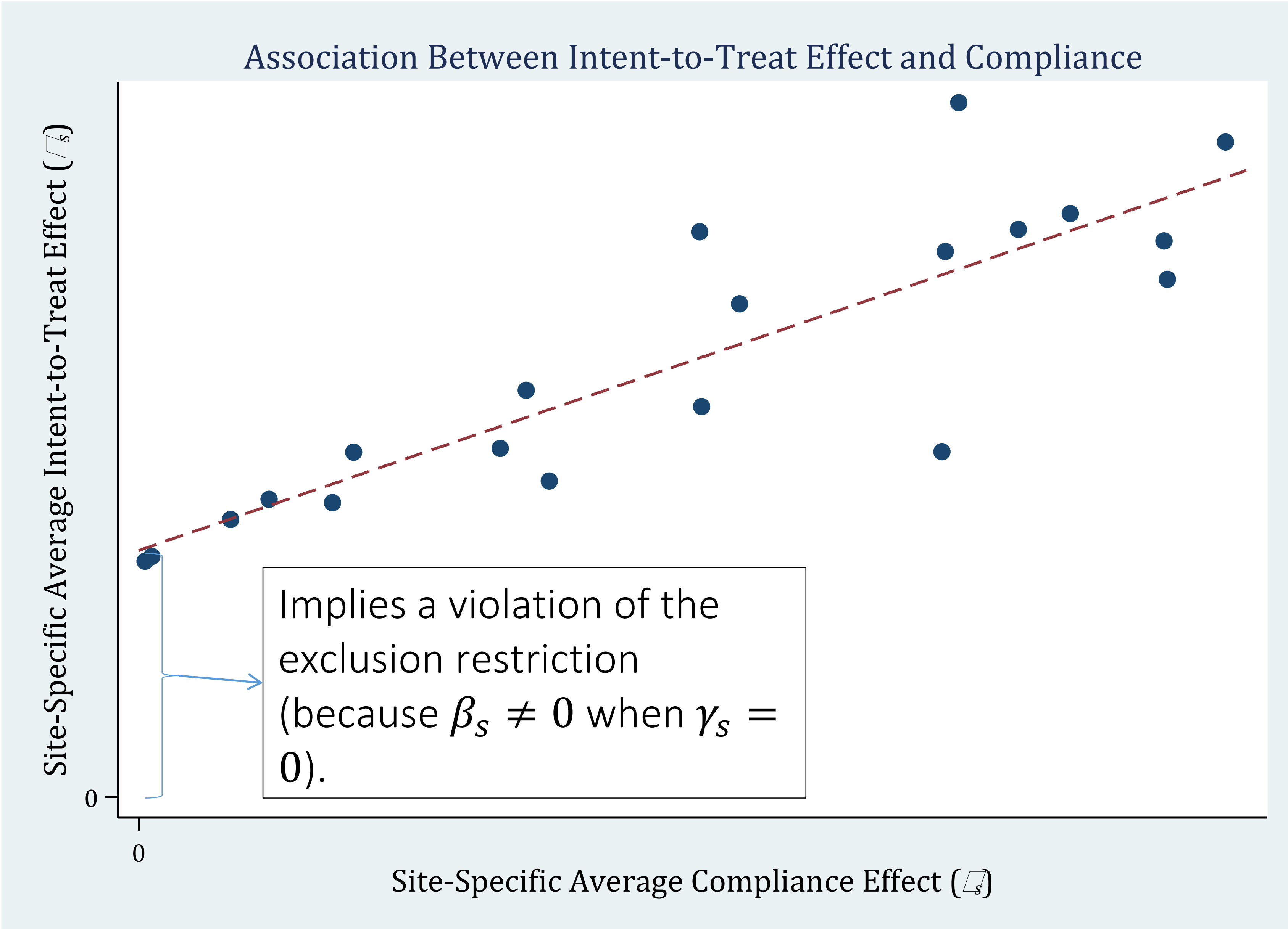

Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect ($_s$)
0
0
Site-Specific Average Compliance Effect ($_s$)



Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect (β_s)
0
0
Site-Specific Average Compliance Effect (γ_s)
γ_s
$\delta_s = \frac{\beta s}{\gamma_s}$
β_s


Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect (β_s)
0
0
Site-Specific Average Compliance Effect (γ_s)
β_s
γ_s
$\delta_s = \frac{\beta s}{\gamma_s}$


Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect ($\square_s$)
0
0
Site-Specific Average Compliance Effect ($\square_s$)
δ


Association Between Intent-to-Treat Effect and Compliance
Site-Specific Average Intent-to-Treat Effect ($_s$)
0
0
Site-Specific Average Compliance Effect ($_s$)
Implies a violation of the exclusion restriction (because $\beta_s \neq 0$ when $\gamma_s = 0$).

Design issues (some intuition):

Precision of estimates of $\hat{\delta}$'s will depend on

- Means and variances of the γ_{ps}'s (more precision when means and variances are larger)
- Variances/covariances of the δ_{ps}'s (more precision with smaller variances and covariances)
- Covariances among γ_{ps}'s (negative covariances yield greater precision)
- Number of sites (J)
- Precision of the $\hat{\gamma}_{ps}$'s (which depend on n_s's and within-site variance in mediators)

Is this design/model useful?

Maybe not…

- It relies on a large number of assumptions
- Need good theory/prior evidence to justify
- May need large number of sites

However…

- Designed variation in mediator impacts can provide a lot of leverage (e.g. Duncan, Morris, Rodrigues 2011).
- Bias due to compliance-effect non-independence may not be large, and can be corrected (Reardon, Unlu, Zhu, & Bloom, 2014).
- Exclusion restriction can be (partially) tested.

The Multiple-site, Multiple-Mediator Instrumental Variables Model (MSMM-IV): Selected Papers

Duncan GJ, Morris PA, Rodrigues C. 2011. Does Money Really Matter? Estimating Impacts of Family Income on Young Children's Achievement with Data from Random-Assignment Experiments. *Developmental Psychology* 47: 1263-79

Kling JR, Liebman JB, Katz LF. 2007. Experimental Analysis of Neighborhood Effects. *Econometrica* 75: 83-119

Nomi T, Raudenbush SW. 2013. Academic Differentiation, Instructional Reform, and Inequality: Evidence from a Natural Experiment in 60 Urban High Schools. Working paper.

Reardon SF, Raudenbush SW. 2013. Under What Assumptions do Site-by-Treatment Instruments Identify Average Causal Effects? *Sociological Methods and Research.*

Reardon SF, Unlu F, Zhu P, Bloom H. 2013. Bias and Bias Correction in Multi-Site Instrumental Variables Analysis of Heterogeneous Mediator Effects. *Journal of Educational and Behavioral Statistics.*