Causal Mediation Analysis

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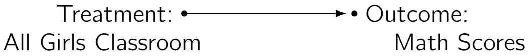
Overview

- This talk is based on a series of papers with Kosuke Imai, Dustin Tingley, and Teppei Yamamoto

Papers are at `http://www.personal.psu.edu/ljk20`

Experiments and Causal Inference

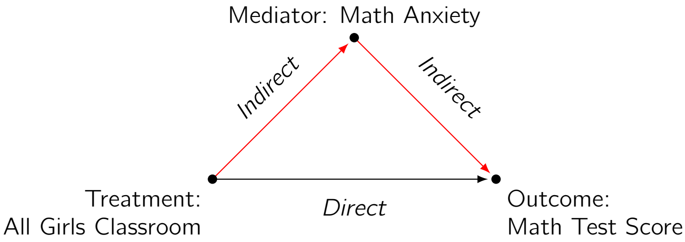


WHY did the treatment work?

Deaton (2010): it's a "black-box."

Causal mediation analysis

- Shapka & Keating (2003): math anxiety.



Total Effect = Indirect + Direct

Standard Approach to Mediation Analysis

Linear structural equation models:

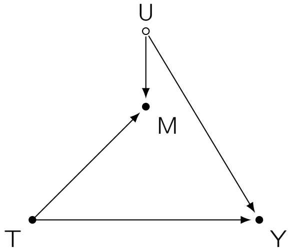
$$
\begin{aligned}
Scores_i &= \alpha_1 + \beta_1 Treat_i + \epsilon_{1i}, \\
Anxiety_i &= \alpha_2 + \beta_2 Treat_i + \epsilon_{2i}, \\
Scores_i &= \alpha_3 + \beta_3 Treat_i + \gamma Anxiety_i + \epsilon_{3i}
\end{aligned}
$$

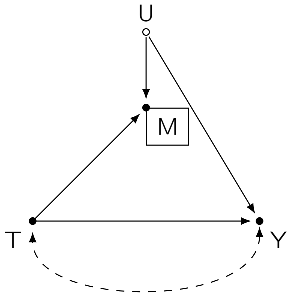
- Total effect (Average Treatment Effect): β_1.
- Direct effect: β_3.
- Indirect (mediated) effect: $\beta_2\gamma$.
- Total effect: $\beta_1 = \beta_3 + (\beta_2\gamma)$.

The Complication: Conditioning on a Post-Treatment Quantity

$$
\begin{aligned}
Scores_i &= \alpha_1 + \beta_1 Treat_i + \epsilon_{1i} \\
Scores_i &= \alpha_3 + \beta_3 Treat_i + \gamma Anxiety_i + \epsilon_{3i}
\end{aligned}
$$

Conditioning on a Post-Treatment Quantity





Potential Outcomes Notation for Causal Mediation Analysis

Observed quantities:

- Binary treatment: $T_i \in \{0, 1\}$, all-girls classroom
- Mediator: M_i, math anxiety
- Outcome: Y_i, math test scores
- Observed pre-treatment covariates: X_i.

The potential outcomes:

- Potential mediators: $M_i(t)$
- Potential outcomes: $Y_i(t, m)$

Example with this notation

$M_i(1)$ is the **observed** level of math anxiety reported by individual i, who was assigned to an all girls classroom.

$Y_i(1, M_i(1))$ is the **observed** math performance reported by individual i, who was assigned to an all girls classroom, and had a anxiety level $M_i(1)$ which is observed under an all girls classroom.

$M_i(0)$ and $Y_i = Y_i(0, M_i(0))$ are the converse.

Total Effect

- Total effect = ATE:

$$\tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0))$$

- Randomization of the treatment makes the following true:

  $Y_i(1, M_i(1)), Y_i(0, M_i(0)) \perp\!\!\!\perp T_i$

Indirect or mediation effects

- Causal mediation (indirect) effects:

$$\delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))$$

- Change the mediator from $M_i(0)$ to $M_i(1)$ while holding the treatment constant at t.

The Counterfactual

- $t_i = 1$:

  $Y_i(1, M_i(1))$ *is the observed math score if student is assigned to an all girls classroom, with an anxiety level produced by the all girls class.*

- $t_i = 1$:

  $Y_i(1, M_i(0))$ *is her math score in the counterfactual world where subject i was assigned to an all girls classroom but her math anxiety level is at the same level as if she were in a co-ed classroom.*

- "Identification problem:" we don't observe $Y_i(1, M_i(0))$

Identification under Sequential Ignorability

- Necessary identification assumption: Sequential Ignorability (Imai, Keele, Yamamoto, 2010)

$$\{Y_i(t', m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i = x,$$

$$Y_i(t', m) \perp\!\!\!\perp M_i(t) \mid T_i = t, X_i = x$$

Unpacking Sequential Ignorability in the Example

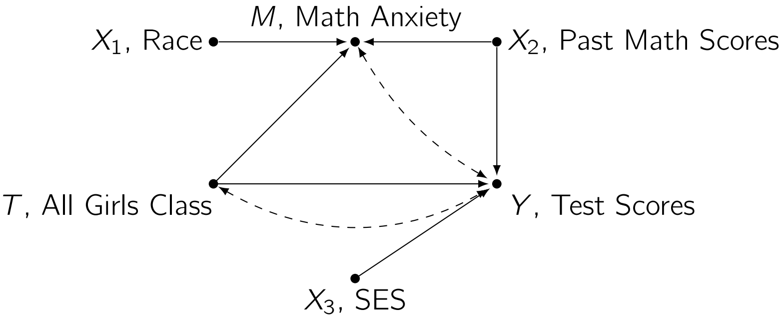


Figure : We can rule out some confounders but not others when treatment is randomized.

Sequential ignorability in this example

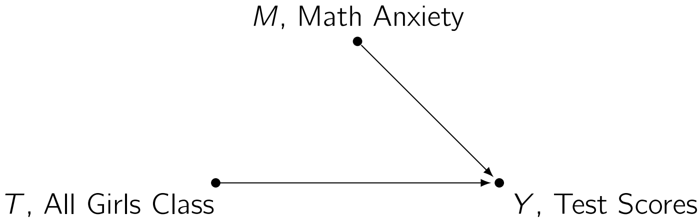
Sequential Ignorability is satisfied if:

- *There are no unobserved pre-treatment covariates that influence both treatment and math anxiety or math scores.*
- *There are no unobserved pre-treatment covariates that influence both math anxiety and math scores.*

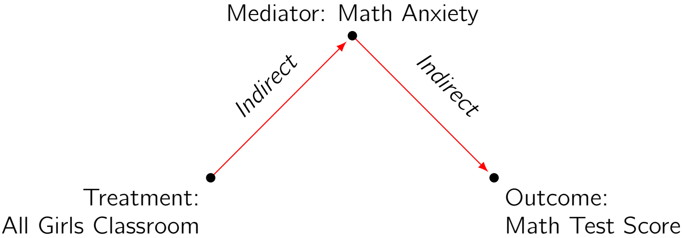
Sequential ignorability

- Untestable
- Must defend your specification

Randomizing The Mediator



IV and mediation analysis



IV: Direct Effect is zero.

Other Issues in Mediation Analysis

- Estimation
- Sensitivity analysis
- Multiple mediators
- Noncompliance

Guidelines

- Articulate and evaluate assumptions: Present total and indirect effects separately
- Measure possible confounders: baseline mediators and outcomes especially.
- Perform a sensitivity analysis

Standard Estimation Methods and Sequential Ignorability

Standard Equations for Mediator and Outcome:

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i}, \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i} \end{aligned}$$

Under sequential ignorability, the average causal mediation effect (ACME) is identified as $\bar{\delta}(0) = \bar{\delta}(1) = \beta_2\gamma$.

Product of coefficients is only a valid measure of the causal mediation effect when:

The M and Y models are linear regressions.

Problem: Linear models are often inappropriate. $\Longrightarrow$ When models aren't linear regressions $\beta_2\gamma$ represents nothing!

Example: Continuous mediator and binary outcome

Suppose

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i}, \\ \Pr(Y_i = 1) &= \Phi\left(\alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i}\right) \end{aligned}$$

Then the ACME is given by

$$\bar{\delta}(t) = \Phi\left(\frac{\alpha_3 + \beta_3 t + \gamma(\alpha_2 + \beta_2)}{\sqrt{\sigma_2^2 \gamma^2 + 1}}\right) - \Phi\left(\frac{\alpha_3 + \beta_3 t + \gamma \alpha_2}{\sqrt{\sigma_2^2 \gamma^2 + 1}}\right)$$

Not $\beta_2 \gamma$.

Estimation algorithm

- Sketch of the algorithm
  - Step 1: Fit some model for the mediator $g(M_i|T_i, X_i)$ and outcome variable $f(Y_i|T_i, M_i, X_i)$
  - Step 2: Predict values of the mediator under both values of the treatment.
  - Step 3: Predict values of the outcome variable under the simulated values of the mediator but using both $T_i = 1$ and $T_i = 0$
  - Step 4: Take the average difference in these two predictions
  - Confidence intervals using bootstrapping.

Example: Continuous mediator and binary outcome

Estimate the two following models:

$$
\begin{aligned}
M_i &= \alpha_2 + \beta_2 T_i + X_i + \epsilon_{2i}, \\
\Pr(Y_i = 1) &= \Phi\left(\alpha_3 + \beta_3 T_i + \gamma M_i + X_i + \epsilon_{3i}\right)
\end{aligned}
$$

- Predict M_i for $T_i = 1$ and $T_i = 0$. This gives you $\hat{M}_i(1)$ and $\hat{M}_i(0)$.
- Predict Y_i with $T_i = 1$ and $\hat{M}_i(0)$ and vice versa.
- Take average of these two predictions.

Sensitivity Analysis

- Sequential ignorability is a unrefutable strong assumption: can't be tested with any configuration of the data.
- Need to assess the robustness of findings via a sensitivity analysis
- A sensitivity analysis asks: how large a departure from the key assumption must occur for the conclusions to no longer hold?
- Statistical interpretation: Parametric sensitivity analysis by assuming

$$\{Y_i(t', m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i = x$$

but not

$$Y_i(t', m) \perp\!\!\!\perp M_i \mid T_i = t, X_i = x$$

Sensitivity Analysis

Sensitivity Analysis:

- Let the resulting correlation in error terms be $\rho = corr(\epsilon_{2i}, \epsilon_{3i})$.
- If $\rho = 0$ sequential ignorability holds. Nonzero values of ρ represent departures from the key assumption.
- We can calculate mediation effect as a function of ρ and observe for what value of ρ, the mediation effect and its confidence interval includes zero.

The Product of Coefficients Method

- The β_2 term represents the magnitude of the relationship between the treatment and the mediator.
- The γ term represents the magnitude of the relationship between the mediator and dependent variable after controlling for the effect of the independent variable.
- The product of these two quantities is the amount of variance in the dependent variable that is accounted for by the independent variable through the mechanism of the mediator.