Bayesian Inference for Sample Surveys

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Distinctive features of survey inference

1. Primary focus on descriptive finite population quantities, like overall or subgroup means or totals
   - Bayes – which naturally concerns predictive distributions -- is particularly suited to inference about such quantities, since they require predicting the values of variables for non-sampled items
   - This finite population perspective is useful even for analytical quantities:

θ = model parameter (meaningful only in context of the model)

$\tilde{\theta}(Y)$ = "estimate" of θ from fitting model to whole population Y

(a finite population quantity, exists regardless of validity of model)

A good estimate of θ should be a good estimate of $\tilde{\theta}$

(if not, then what's being estimated?)

Distinctive features of survey inference

2. Analysis needs to account for "complex" sampling design features such as stratification, differential probabilities of selection, multistage sampling.
   - Samplers reject theoretical arguments suggesting such design features can be ignored if the model is correctly specified.
   - Models are always misspecified, and model answers are suspect even when model misspecification is not easily detected by model checks (Kish & Frankel 1974, Holt, Smith & Winter 1980, Hansen, Madow & Tepping 1983, Pfeffermann & Holmes (1985).
   - Design features like clustering and stratification can and should be explicitly incorporated in the model to avoid sensitivity of inference to model misspecification.

Distinctive features of survey inference

3. A production environment that precludes detailed modeling.
   - Careful modeling is often perceived as "too much work" in a production environment (e.g. Efron 1986).
   - Some attention to model fit is needed to do any good statistics
   - “Off-the-shelf" Bayesian models can be developed that incorporate survey sample design features, and for a given problem the computation of the posterior distribution is prescriptive, via Bayes Theorem.
   - This aspect would be aided by a Bayesian software package focused on survey applications.

Distinctive features of survey inference

4. Antipathy towards methods/models that involve strong subjective elements or assumptions.
   - Government agencies need to be viewed as objective and shielded from policy biases.
   - Addressed by using models that make relatively weak assumptions, and noninformative priors that are dominated by the likelihood.
   - The latter yields Bayesian inferences that are often similar to superpopulation modeling, with the usual differences of interpretation of probability statements.
   - Bayes provides superior inference in small samples (e.g. small area estimation)

Distinctive features of survey inference

5. Concern about repeated sampling (frequentist) properties of the inference.
   - Calibrated Bayes: models should be chosen to have good frequentist properties
   - This requires incorporating design features in the model (Little 2004, 2006).

Survey Inference Setup

$Z = (Z_1, ..., Z_N) =$ design variables, known for population

$Y = (Y_1, ..., Y_N)$ = population values,
recorded only for sample

$Q = Q(Y, Z)$ = target finite population quantity

$I = (I_1, \ldots, I_N)$ = Sample Inclusion Indicators

$$I_i = \begin{cases} 1, & \text{unit included in sample} \\ 0, & \text{otherwise} \end{cases}$$

$Y_{\text{inc}} = Y_{\text{inc}}(I) =$ part of Y included in the survey

$Y = (Y_{\text{inc}}, Y_{\text{exc}})$

I	Z	Y
1		
1		Y_{inc}
1		
0		
0		
0		$[Y_{\text{exc}}]$
0		
0		

Models

- Joint distribution of *(Y, I)* conditional on *Z*
- Two approaches

$$\Pr(Y, I \mid Z) = \Pr(Y \mid Z)\Pr(I \mid Y, Z)$$

$$\Pr(Y, I \mid Z) = \Pr(Y \mid I, Z)\Pr(I \mid Z)$$

- Typically

(Sampling mechanism does not depend on the survey outcomes) $\rightarrow$ $\Pr(I \mid Y, Z) = \Pr(I \mid Z)$

(Same substantive model applies to both sampled and nonsampled Subjects) $\rightarrow$ $\Pr(Y \mid I, Z) = \Pr(Y \mid Z)$

Model Specification

- Indices used to identify subjects in the population (conditional on Z) is assumed to be arbitrary
- Exchangeable joint distribution

$$\Pr(Y_1, Y_2, \cdots, Y_N \mid Z) \equiv \Pr(Y_{i_1}, Y_{i_2}, \cdots, Y_{i_N} \mid Z)$$

$$(i_1, i_2, \cdots, i_N) \text{ is a permutation of } (1, 2, \cdots, N)$$

- Exchangeable distribution are of the form

$$Y_i \mid Z, \theta \sim independent$$
$$\pi(\theta) \equiv prior$$

Examples

- Assume SRS and no Z, binary Y

$$Y_i \mid \theta \sim iid\ \ Bern(1,\theta), i = 1, 2, \cdots, N$$
$$\theta \sim Beta(a,b); a, b\ known$$

- Z: H Strata, SRS within stratum, Continuous Y

$$Y_{ih} \mid Z = h \sim iid\ \ N(\mu_h, \sigma_h^2)$$
$$\pi(\mu_h, \log\sigma_h) \sim BVN$$

- Cluster sampling, Count Y

$$Y_{ic} \sim iid\ \ Poisson(\lambda_c), i = 1, 2, \cdots, N_c$$
$$\log\lambda_c \sim iid\ \ N(\mu, \sigma^2), c = 1, 2, \cdots, C$$
$$\pi(\mu, \log\sigma) \sim BVN$$

Inference

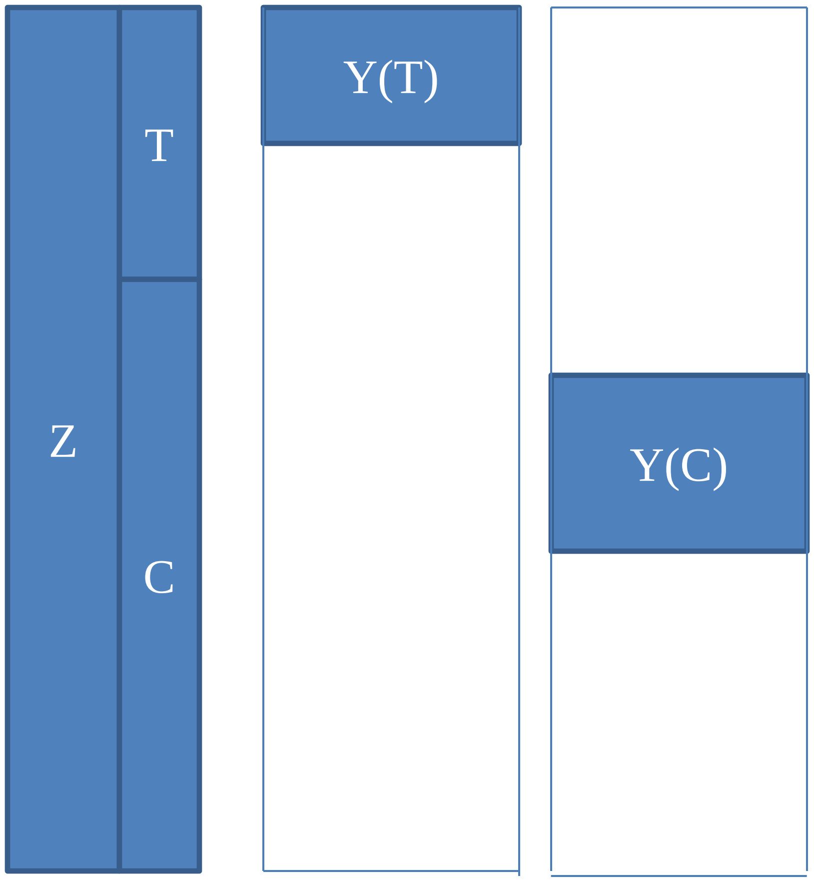
- Observed data: $\{Y_{inc}, Z, I\}$
- Unobserved or missing data: Y_{exc}
- Model: $\Pr(Y \mid Z)$
- Inference: $\Pr(Y_{exc} \mid Z, I, Y_{inc})$
- Goal: Simulate copies of Y_{exc} by drawing from the above predictive distribution and compute the estimand of interest *Q(Y,Z)*
- Multiple Imputation of Missing Values or create synthetic populations

Example

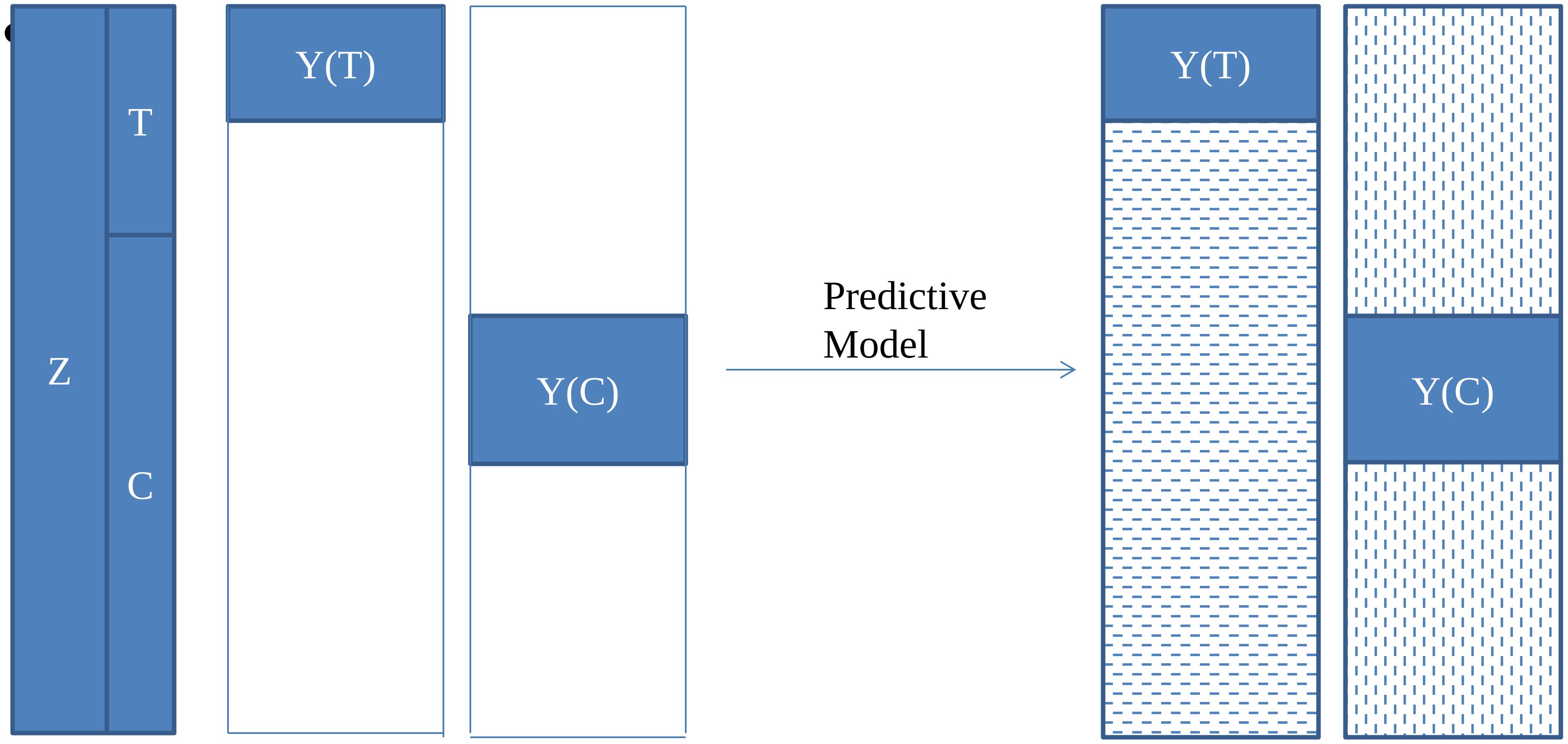
- Housing and Children Study to evaluate the effect of providing housing voucher on child development
- Population: All applicants for voucher
- Treatment: Random Selection
- Control: Rest of the population
- Survey: Samples of Treatment and Control subjects
- Two waves, Dried Blood spots, Child development measures, adult primary care giver

Data Setup

- Z: Data from sampling frame (from voucher application)
- T for Treatment and C for Control
- Y(T): Measures for Treatment subjects
- Y(C): Measures for Control Subjects



Fill-in Synthetic potential populations



Inference

- Create several potential synthetic populations under treatment and control conditions
- Compute summary measures (such as mean, median etc.)
- Compare the distribution of summary measures under treatment and control conditions
  - Numerical summaries
  - Graphical summaries like histogram or kernel densities
- Analyze the two sets of populations to discern treatment effects, heterogeneity of treatment effects etc.

Summary

- Bayes inference for surveys must incorporate design features such as stratification, weighting and clustering appropriately
- Bayes inference is not asymptotic, and delivers good frequentist properties in small samples
- Software like BUGS (PROC MCMC in SAS) can be used to implement fully model based framework
- Recasting the Bayesian inference problem as missing data problem allows the use of multiple imputation software
- Nonparametric Bayes allows incorporation of complex design features without making strong model assumptions
- Pseudo or synthetic population framework makes the inference problem easy (just compute any estimand of interest)
- Give it a try!! (you will love it ☺)